Comparative Study of Distinguishing Sets in Graphs

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Abstract

Consider safeguard analysis of a facility, modeled as a graph where detecting devices are placed at its vertices with the connections between vertices as the edges. The detectors are assumed to send signals to the control center when they detect an intruder or hazard such as fire. Here we are interested in identifying the exact location of an intruder by placing the minimum number of detectors.

Depending on the capabilities of the detector, various graphical parameters have been defined and these are called distinguishing sets. Three well-known distinguishing set parameters include Identifying Codes, Locating-Dominating Sets, and Open-Locating-Dominating Sets and the problems of determining these sets are known to be NP-complete for an arbitrary graph.

In this paper we compare and summarize the properties and previous results on these parameters. We also implement a program to determine the minimum values of these parameters for certain classes of graphs.

Introduction

Graphs represent relationships from most abstract to most concrete. Since they have powerful abstractions, graphs can be very important in modeling data. They have wide applications in different areas like Computer Science, Physics and Chemistry, Social Science, Mathematics, Biology etc. One such application of graph is safeguard analysis of a facility.

For graph G= (V,E) be a graph and V={1,,...,n}:

- The **open neighborhood** of a vertex u is the set N(u) of all vertices of G adjacent to u.
- N[u] ={u} ∪ N(u) is the **closed neighborhood** of u.

Dominating Set: In graph theory, a dominating set for a graph G= (V,E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D [10]. That is, a subset D ⊆ V is dominating if N[i] ∩ C is non-empty set for all i ∈ V.
Size of dominating set is about one-third of total number of vertices. Figure 1, line 1 shows the dominating set with dark vertices and the red codes below the graph is the set of vertices dominating that vertex.

NP-complete problem: NP stands for "non-deterministic polynomial time" which means that the problem can be solved in polynomial time with a special non-deterministic algorithm.

Methods and Materials

The vertices dominating every vertex in the graph should be unique for the following sets. This is not true for the dominating set.:

1. Identifying Code (IC): Half of the total vertices in the graph make up for the identifying code set.

2. Locating-dominating(LD) set: It is assumed that the vertices in this set can distinguish themselves from their neighbors. Two-fifth of the total vertices make up for the LD set

3. Open Locating-dominating Set: In this set the vertex cannot dominate itself and requires other vertices to dominate itself, thus increasing the size of OLD set to about two-third of the total vertices.

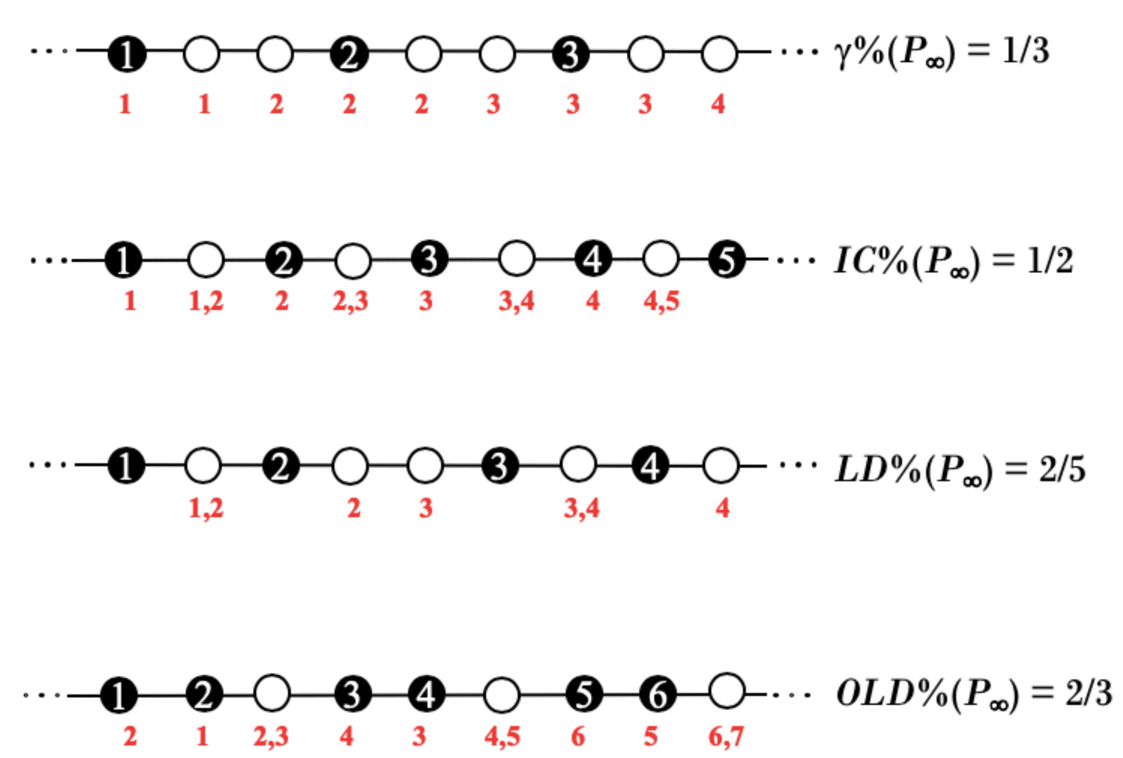

Figure 1. The dominating, IC, LD and OLD sets for infinite path graph

Graph class	ID	LD	OLD
Bipartite	NP-c	NP-c	NP-c
Chordal bipartite	NP-c	NP-c	NP-c
Planar max. degree 3	NP-c	NP-c	OPEN
Planar bipartite max. degree 3	NP-c	NP-c	NP-c
(Planar) line	NP-c	OPEN	OPEN
Planar bipartite unit disk	NP-c	NP-c	NP-c
Bounded tree-width/clique-width	P	P	P
Line of bounded tree-width	P	P	P
Split	NP-c	NP-c	Np-c

Table 1. Label in 24pt Calibri.

Results

- The problems of finding minimum IC, LD and OLD set are in general NP-complete for most graph classes. Table 1 summarizes the computational complexity of these problems for some graph classes.
- Reviewed the NP-complete proof for the problem of finding minimum size of OLD set: The problem was proved NP-complete by reducing 3-SAT to OLD in polynomial time. Figure 2(a) is a graph with 21 vertices and 2(b) is a graph with 7 vertices. Suppose we complete graph G by connecting clause C_j to (ui,1, ui,2, ui,3) where u is u or $\overline{u}$ and S ⊆ V(G) is any OLD(G)-set, then vertex a_j is covered by b_j or $N(a_j) \cap S = b_j$. So $N(c_j) \cap S = u_{i,1}$ or $u_{i,2}$ or $u_{i,3}$. Because $N(a_j) \cap S \neq N(c_j) \cap S$. So $S \cap \{u_{1,j}, u_{2,j}, u_{3,j}\} \neq Ø$. This proves that C has a satisfying truth assignment if and only if OLD(G) = (9N + 3M) + N = 10N + 3M [8].
- Implemented a program in Python programming language using NetworkX package to find minimum size of dominating, IC, LD and OLD set for a finite path graph. This program takes number or vertices as an input, constructs a path graph and determines the distinguishing sets for that graph. Figure 3 and 4 shows the output for the path graph for 13 and 7 number of vertices, respectively.

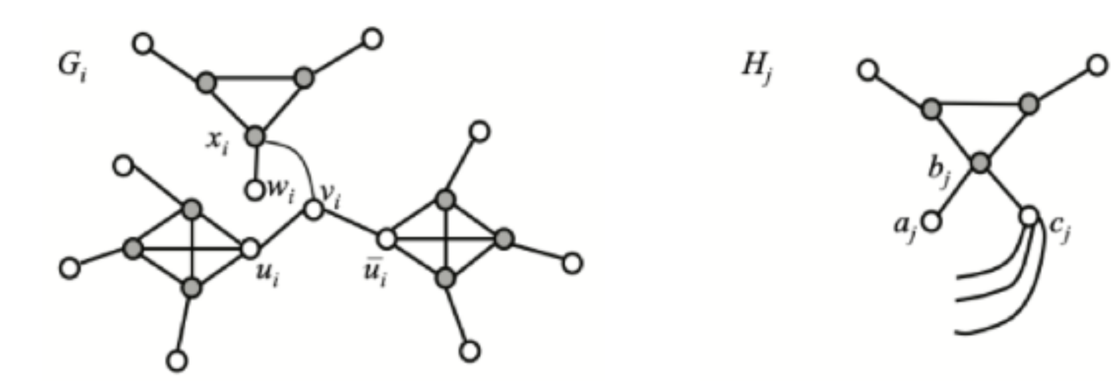

Figure 2. OLD problem is NP-complete

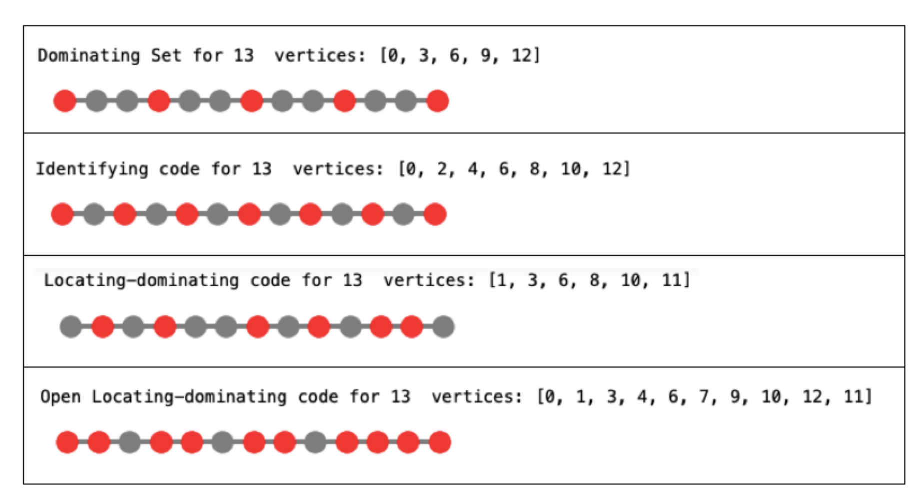

Figure 3. Program output for path graph with 13 vertices

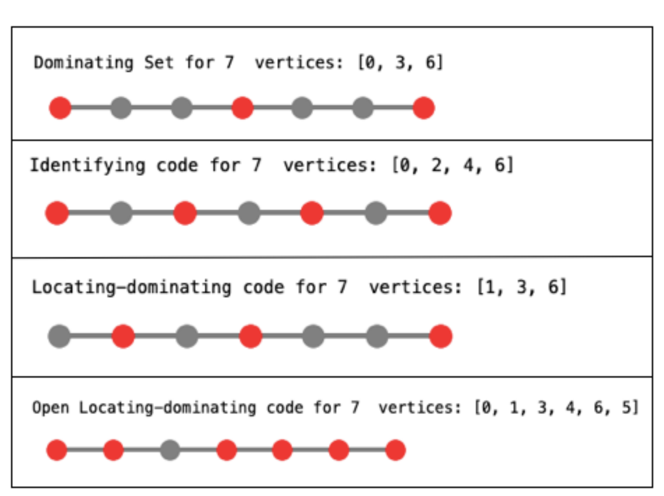

Figure 4. Program output for path graph with 7 vertices

Conclusions

Studied and compared the important distinguishing sets in graphs. Reviewed the NP-completeness nature and proof for the problem of minimum size of IC, LD and OLD problems. Implemented a program to determine these distinguishing sets for the finite paths.

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